## The impact of current harmonics on the power factor of a low voltage distribution system - A Case Study

## Introduction

This study was carried out to identify and rectify, what initially seemed to be a malfunction of a power factor correction capacitor panel (PFCCP). However during the study it was identified that the problem was not due to a malfunction of the PFCCP, but due to the failure to identify the impact of the harmonics on the power factor, at the design stage. In this case study we will analyze the problem and try to find out a theoretical explanation comparing the data gathered at site.

Kekirawa water supply scheme has two 132 kW pumps to pump the water from Yodha Ela to the water treatment plant. The two pumps are controlled by two Variable Speed Drives rated at 132 kW working in duty/standby basis. There is a Motor Control Panel Board located inside the pump house consisted of the VSDs and other control and switch gear. There is a separate Main Incoming and Distribution Panel Board which supplies power to the Motor Control Panel and auxiliary power to the pump house.
Auxiliary power of the pump house consist of four florescent lights, two socket outlets, an overhead crane, indication bulbs and exhaust fans of the panel boards. As the crane is used only for a maintenance work of the pumps, and the lighting and socket outlet power is negligible comparing to the pump power, it is safe to assume that the total power consumption of the installation at any given moment is 132 kW . Also this load is a nonlinear load supplied through a VSD.

There is a Power Factor Correction Capacitor Bank integrated in the Main Incoming and Distribution Panel. We were informed that the Power Factor Correction Capacitor Bank is not correcting the power factor as expected and it has not reduced the electricity bill.

## Problem

At the time of measurement one VSD was running at full load which is 132 kW
According to the power analyzer the power factor of the system is shown as 0.85
Needed capacitive reactive power to bring up the power factor to 1 is 81.81 kVar as shown in the below calculation


$$
\begin{aligned}
& \mathrm{Q}_{\mathrm{c}}=P \times\left(\tan \varnothing_{2}-\tan \varnothing_{1}\right) \\
& \text { Where } \mathrm{Q}_{\mathrm{c}}=\text { Needed Capacitive Power } \\
& \varnothing_{2}=\text { Initial power factor angle } \\
& \emptyset_{1}=\text { Desired power factor angle } \\
& \mathrm{Q}_{\mathrm{c}}=132 \times\left(\tan \left(\cos ^{-1}(.85)\right)-\tan \left(\cos ^{-1}(1)\right)\right) \\
&=132 \times(\tan (31.79)-\tan (0)) \\
&=132 \times .42 \\
&=81.81 \mathrm{kVar}
\end{aligned}
$$

Accordingly, it is expected to see one 25 kVar step and one 50kVar step activated in this situation.
However, actually there wasn't any capacitor steps activated.
Also the power factor regulator (Lovato-DCRK 12) recognizes power factor to be around 0.99 which contradicts the power analyzer reading.

It is understood that the reason for not activating the capacitor bank steps is because the power factor regulator reading the power factor as 0.99 . But why is the power factor regulator reading differs from the power analyzer reading? Probable causes can be faulty wiring or power factor regulator malfunctioning. However the results were same even after the wiring was rechecked and power factor regulator was replaced with a new one.

Capacitor steps were activated manually to see if it will improve the power factor. Following table shows the readings taken for different number of capacitor steps activated. It is noticeable how the addition of capacitor steps does not affect the power factor shown by the power analyzer until the $3^{\text {rd }}$ step is added. However after the addition of $3^{\text {rd }}$ step the power factor suddenly jumped from 0.87 lagging to 0.85 Leading.

Table 1. - Comparison between Power Analyzer and Power Factor Regulator readings

| Cap Bank Step |  | Power Quality Analyzer - PW3360-21 |  |  |  | Power <br> Factor <br> Regulator <br> DCRK 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| No | kVar (Actual) | kW | kVA | kVar | PF | PF |
| 0 | 0 | 95.1 | 110.4 | 56.9 | 0.86 | 0.98 |
| 1 | 7.2 | 95.5 | 109.9 | 55.6 | 0.86 | 0.99 |
| 2 | 7.2 | 95.2 | 109.3 | 54.7 | 0.87 | 1 |
| 3 | 34.6 | 95.6 | L-113.6 | L-60.2 | L-0.85 | 0.98 (Lead) |

All these observations does not agree with the theoretical relation between the active power, reactive power and the power factor.

For example let's take the situation where the $3^{\text {rd }}$ step was activated adding 34.6 kVar to the system. If we consider the power analyzer readings, let's see what should be the resulting power factor, after adding 34.6 kVar capacitor step.

$$
\text { Qc = 34.6kVar, P1 ~ P2 = 95.4kW, PF } 1=0.87
$$



```
\(Q_{R}=P x \tan \left(\varnothing_{1}\right)-Q_{C}\)
    \(=95.4 \times \tan \left(\cos ^{-1}(0.87)\right)-34.6\)
    \(=19.47 \mathrm{kVar}\)
\(\varnothing_{2}=\operatorname{Tan}^{-1}\left(Q_{R} / P\right)\)
    \(=\operatorname{Tan}^{-1}(19.47 / 95.4)\)
    = 11.54
```

$\varnothing_{2}$ does not become negative means the power factor does not become leading.
$P F_{2}=\operatorname{Cos}(11.54)=0.97$ Lagging
However according to the observation $\mathrm{PF}_{2}=0.85$ Leading
Searching for an answer for this contradicting results we have stumbled upon a common mistake done by most people when dealing with power factor correction. The common misconception is,

Power factor $=\operatorname{Cos} \varnothing ; \quad$ where $\varnothing$ is the phase angle between voltage and current Waveforms
The above expression is not always true and we have shown the theoretical explanation in the following section.

## Theoretical explanation

The expression for the power factor in the above section is true only for the special case where the voltage and current waveforms are purely sinusoidal. For this to happen the loads should be linear without having high speed switching components. Let's derive the expression for the power factor considering this special case,

Definition of the power factor;
$p f=$ Pavg $/ S=$ Pavg $/$ Vrms.Irms

Where, $\quad P_{\text {avg }}=A v e r a g e ~ A c t i v e ~ p o w e r, ~$ $\mathrm{S}=$ Apparent power
$\mathrm{V}_{\mathrm{rms}}=$ Root Mean Square value of voltage waveform
$I_{\text {rms }}=$ Root Mean Square value of current waveform

For pure sinusoidal waveforms;
$V(t)=V p \operatorname{Sin}(\omega t+\emptyset 1)$
$I(t)=I p \operatorname{Sin}(\omega t+\emptyset 2)$
Where, $\quad V_{P=P}$ Peak value of voltage,
$I_{p}=$ Peak value of voltage,
$\emptyset_{1}=$ Phase angle of Voltage waveform,
$\emptyset_{2}=$ Phase angle of Current Waveform

$$
\begin{aligned}
P(t)= & V(t) \cdot I(t) \\
P(t)= & V p X \operatorname{Ip} X \operatorname{Sin}(\omega t+\emptyset 1) x \operatorname{Sin}(\omega t+\emptyset 2) \\
& =V p X \operatorname{Ip} X 1 / 2\{-\operatorname{Cos}(\omega t+\emptyset 1+\omega t+\emptyset 2)+\cos (\omega t+\emptyset 2-(\omega t+\emptyset 1))\} \\
& \left.=\left(\frac{V p}{\sqrt{2}}\right) X\left(\frac{I p}{\sqrt{2}}\right) X\{-\operatorname{Cos}(2 \omega t+\emptyset 1+\emptyset 2)+\cos (\varnothing 1-\emptyset 2))\right\}
\end{aligned}
$$

Let's discard the sinusoidal portion of the above expression to get the average value of active power
$\operatorname{Pavg}=\frac{V p}{\sqrt{2}} X \frac{I p}{\sqrt{2}} \times \operatorname{Cos}(\varnothing 1-\emptyset 2)$
Pavg $=$ Vrms $x$ Irms $X \operatorname{Cos}(\emptyset 1-\emptyset 2)$
In the same way we can show that,

$$
\text { Qavg }=\operatorname{Vrms} x \operatorname{Irms} \quad X \operatorname{Sin}(\emptyset 1-\emptyset 2)
$$

By definition of the apparent power;
$S=$ Vrms $x$ Irms
From 1) and 4) we can get
$p f=V r m s x \operatorname{Irms} X \operatorname{Cos}(\emptyset 1-\emptyset 2) / V r m s x \operatorname{Irms}$
$p f=\operatorname{Cos}(\emptyset 1-\emptyset 2)$
Where; $\quad \emptyset 1$ - $\emptyset 2=\emptyset$; the phase angle difference between voltage and current waveforms
This means the expression,
$p f=\operatorname{Cos}(\varnothing) ;$
Is true when the voltage and current waveforms are purely sinusoidal. We call this "Displacement power factor" because it is caused by the displacement of the voltage and current waveforms.

Now let's see what happens when the voltage and current waveforms are not purely sinusoidal. In many practical situations these waveforms can be deformed due to various nonlinear loads such as VSDs, UPS and other high speed switching devices.

Any deformed (Non sinusoidal) periodic waveform can be expressed as a summation of a series of sinusoidal waveforms with different frequencies. The sinusoidal waveform with the frequency equal to the original waveforms frequency is called the fundamental waveform and the other waveforms will have frequencies that are integer multiplies of the fundamental waveforms frequency. Those waveforms are called harmonics.

Expressions for voltage and current with the harmonics become,
$V(t)=\sum_{k=1}^{\infty} V k \operatorname{Sin}(k \omega t+\delta k)$
$I(t)=\sum_{k=1}^{\infty} I k \operatorname{Sin}(k \omega t+\theta k)$

Where; $\quad V k=$ Peek value of the voltage waveform of the $k^{\text {th }}$ harmonic $\mathrm{Ik}=$ Peek value of the current waveform of the $\mathrm{k}^{\text {th }}$ harmonic $\delta k=$ Phase angle of the voltage waveform of the $k^{\text {th }}$ harmonic $\theta \mathrm{k}=$ Phase angle of the current waveform of the $\mathrm{k}^{\text {th }}$ harmonic

For these waveforms r.m.s values can be shown as,
Vrms $=\sqrt{\sum_{k=1}^{\infty} V_{k r m s}^{2}}$
$\operatorname{Irms}=\sqrt{\sum_{k=1}^{\infty} I_{k r m s}^{2}}$

The Average Power is,

Pavg $=\sum_{k=1}^{\infty} V_{k r m s} I_{k r m s} \operatorname{Cos}\left(\delta_{k}-\theta_{k}\right)=P_{1 a v g}+P_{2 a v g}+P_{3 a v g}+\cdots$

Normally the level of the harmonics existing in a waveform is expressed by the Total Harmonic Distortion (THD). Equations for the THD of voltage and current waveforms are,
$T H D V=\frac{\sqrt{\sum_{k=2}^{\infty} V_{k r m s}^{2}}}{V_{1 r m s}} .100 \%$
$T H D I=\frac{\sqrt{\sum_{k=2}^{\infty} I_{k r m s}^{2}}}{I_{1 r m s}} .100 \%$

By substitution 13) and 14) in 10) and 11);
$V r m s=V_{1 r m s} \sqrt{1+\left(\frac{T H D_{V}}{100}\right)^{2}}$
$I r m s=I_{1 r m s} \sqrt{1+\left(\frac{T H D_{I}}{100}\right)^{2}}$

By Substitution 15) and 16) in 1);
$p f($ True $)=\frac{P_{\text {avg }}}{V_{1 r m s} I_{1 r m s} \sqrt{1+\left(\frac{T H D v}{100}\right)^{2}} \sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}$
$p f($ True $)=\frac{P_{\text {avg }}}{V_{1 r m s} I_{1 r m s}} \cdot \frac{1}{\sqrt{1+\left(\frac{T H D v}{100}\right)^{2}}} \cdot \frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}$

To simplify the above expression we can make the following assumptions;

1. The contributions to the average power by the harmonics above fundamental is very small. This assumption is verified by the Graph 4 which is taken from the readings of power quality analyzer at Kekirawa pump station.

$$
\therefore P_{a v g} \sim P_{a v g 1}
$$



Graph 1. - The contributions to the average power by the harmonics above fundamental is very small
2. THDv is usually below $5 \%$. This assumption is verified by the Table 1 where the THDV is $2.48 \%$.

$$
\therefore V_{r m s} \sim V_{1 r m s}
$$

| Order | Active power (sum) |  |  | Active power (CH1) |  |  | Votage |  |  | Curment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | VL_P |  | LVL_P1 |  |  |  |  |  | LVL_I1 |  |  |
|  | [ kW ]] | [*] | 17 | [1w] | [8] | 17 | [M] | [8] | [1] | (A) | [ 81 | 1 |
| 1 | 104.1 | 100.00 | -11.03 | 31.56 | 100.00 | -2.54 | 228.65 | 10000 | 0.00 | 138.1 | 10000 | -2.54 |
| 2 | 0.0 | 0.00 | 23.92 | -0.00 | -0.00 | -34.14 | 0.09 | 0.04 | -160.57 | 2.5 | 1.79 | -171.50 |
| 3 | -0.0 | -0.02 | 97.30 | -0.00 | -0.01 | 93.58 | 1.58 | 0.69 | 169.44 | 26.5 | 19.06 | .96.68 |
| 4 | -0.0 | -0.00 | -161.14 | -0.00 | -0.00 | -128.24 | 0.07 | 0.03 | -177.95 | 0.5 | 0.37 | -20.45 |
| 5 | 0.5 | 0.53 | 6.79 | 0.19 | 0.58 | -0.30 | 3.03 | 1.33 | 114.93 | 61.0 | 43.57 | 115.53 |
| 6 | 0.0 | 0.00 | .73.12 | 0.00 | 0.00 | -52.76 | 0.03 | 0.01 | 6.37 | 0.6 | 0.41 | -15.19 |
| 7 | -0.1 | -0.07 | 110.42 | -0.01 | -0.05 | 109.52 | 1.38 | 0.60 | 118.40 | 27.2 | 19.59 | -129.24 |
| 8 | -0.0 | -0.00 | 91.70 | -0.00 | -0.00 | 80.03 | 0.08 | 0.04 | -1152.52 | 0.5 | 0.36 | -43.39 |
| 9 | 0.0 | -0.00 | 97.05 | -0.00 | -0.00 | 93.58 | 0.89 | 0.39 | -73.95 | 5.7 | 4.07 | 20.24 |
| 10 | -0.0 | -0.00 | 85.34 | -0.00 | -0.00 | 75.15 | 0.06 | 0.03 | -160.08 | 0.3 | 0.19 | -106.05 |
| 11 | -0.0 | -0.02 | 11250 | -0.01 | -0.03 | 113.18 | 1.82 | 0.80 | -157.92 | 12.6 | 2.04 | -43.93 |
| 12 | 0.0 | -0.00 | 31.97 | -0.00 | -0.00 | -120.46 | 0.04 | 0.02 | -41.71 | 0.3 | 0.19 | -162.17 |
| 13 | 0.0 | 0.00 | 85.64 | 0.00 | 0.00 | 72.85 | 1.15 | 0.51 | -108.80 | 4.1 | 292 | -33.83 |
| 14 | 0.0 | -0.00 | 88.05 | -0.00 | -0.00 | 78.46 | 0.07 | 0.03 | -33.00 | 0.4 | 0.28 | 108.40 |
| 15 | -0, | -0.00 | 95.78 | -0.00 | -0.00 | 95.56 | 0.71 | 0.31 | 58.00 | 3.6 | 259 | 155.11 |
| 16 | -0, | -0.00 | 78.61 | -0.00 | -0.00 | 53.66 | 0.05 | 0.02 | -64.79 | 0.2 | 0.15 | 67.36 |
| 17 | 0.0 | -0.00 | 93.26 | -0.00 | -0.00 | 94.18 | 1.83 | 0.80 | 0.13 | 6.5 | 4.63 | 95.82 |
| 18 | -0, | -0.00 | 51.64 | -0.00 | -0.00 | -56.83 | 0.06 | 0.03 | -149.41 | 0.2 | 0.15 | -62.30 |
| 19 | -0.0 | -0.00 | 9575 | -0.00 | -0.00 | 95.39 | 0.62 | 0.27 | -8.02 | 2.6 | 1.50 | 93.11 |
| 20 | -0.0 | -0.00 | 90.50 | -0.00 | -0.00 | 85.31 | 0.09 | 0.04 | 138.72 | 0.3 | 0.24 | -100.98 |
| 21 | -0.0 | -0.00 | 9203 | -0.00 | -0.00 | 92.35 | 0.75 | 0.33 | -161.56 | 2.5 | 1.79 | .68.06 |
| 22 | -0.0 | -0.00 | 81.41 | -0.00 | -0.00 | 68.52 | 0.05 | 0.02 | 116.02 | 0.2 | 0.13 | -146.98 |
| 23 | 0.0 | -0.00 | 9233 | -0.00 | -0.00 | 92.11 | 1.45 | 0.64 | 146.95 | 4.5 | 321 | -119.81 |
| 24 | -0.0 | -0.00 | 4304 | -0.00 | -0.00 | -62.65 | 0.06 | 0.03 | -9.30 | 0.2 | 0.12 | 73.91 |
| 25 | -0.0 | -0.00 | 92.10 | -0.00 | -0.00 | 84.98 | 0.50 | 0.22 | 136.37 | 1.4 | 1.04 | -133.34 |
| 26 | -0.0 | -0.00 | 92.42 | -0.00 | -0.00 | 91.13 | 0.10 | 0.04 | -50.73 | 0.3 | 0.20 | 44.93 |
| 27 | -0.0 | -0.00 | 91.41 | -0.00 | -0.00 | 90.45 | 0.73 | 0.32 | -25.45 | 1.9 | 1.37 | 67.09 |
| 28 | -0.0 | -0.00 | 85.92 | -0.00 | -0.00 | 51.84 | 0.05 | 0.02 | . 96.84 | 0.1 | 0.11 | 2.17 |
| 29 | -0.0 | -0.00 | 9285 | -0.00 | -0.00 | 91.66 | 1.26 | 0.58 | -69.72 | 3.1 | 220 | 22.89 |
| 30 | -0.0 | -0.00 | 39.09 | -0.00 | -0.00 | -24.63 | 0.06 | 0.03 | -125.90 | 0.1 | 0.10 | -150.53 |
| 31 | -0.0 | -0.00 | 92.10 | -0.00 | -0.00 | 81.25 | 0.34 | 0.15 | -86.97 | 0.8 | 0.53 | 3.03 |
| 32 | -0.0 | -0.00 | 88.50 | -0.00 | -0.00 | 82.53 | 0.10 | 0.04 | 96.24 | 0.2 | 0.17 | -169.60 |
| 33 | -0.0 | -0.00 | 91.56 | -0.00 | -0.00 | 90.19 | 0.74 | 0.32 | 108.40 | 1.5 | 1.10 | -158.01 |
| 34 | -0.0 | -0.00 | 88.87 | -0.00 | -0.00 | 38.08 | 0.04 | 0.02 | 58.29 | 0.1 | 0.08 | -174.73 |
| 35 | 0.0 | -0.00 | 9228 | -0.00 | -0.00 | 91.44 | 1.04 | 0.45 | 73.01 | 2.1 | 1.54 | 164.61 |
| 36 | -0.0 | -0.00 | 44.85 | -0.00 | -0.00 | 19.37 | 0.07 | 0.03 | -105.56 | 0.1 | 0.09 | -14.90 |
| 37 | -0.0 | -0.00 | 9248 | -0.00 | -0.00 | 75.23 | 0.17 | 0.08 | 57.00 | 0.4 | 0.27 | 144.32 |
| 38 | -0.0 | -0.00 | 88.77 | -0.00 | -0.00 | 81.34 | 0.10 | 0.04 | -122.89 | 0.2 | 0.13 | -25.05 |
| 39 | -0.0 | -0.00 | 9248 | -0.00 | -0.00 | 90.89 | 0.71 | 0.31 | -116.53 | 1.3 | 089 | -21.68 |
| 40 | -0.0 | -0.00 | 84.79 | -0.00 | -0.00 | 13.02 | 0.04 | 0.02 | -135.75 | 0.1 | 0.06 | -47.29 |
| THD | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | 2.48 |  |  | 53.21 | $\square$ |

Table 1. - THDV is $\mathbf{2 . 4 8 \%}$

By incorporating the assumptions in 17);
$p f=\frac{P_{1 \text { avg }}}{V_{1 r m s} I_{1 r m s}} \cdot \frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}=p f($ Displacement $) \cdot \frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}$

According to the above equation we can see that there are two components in the power factor. The component other than the displacement power factor is called "Distortion power factor" because it is caused by the distortions in the voltage and current harmonics.
$p f($ distortion $)=\frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}$
Finally we can arrive at the following general equation for power factor which is called the "True power factor"
$p f($ true $)=p f($ Displacement $) \cdot p f($ Distortion $)$
According to this there are two types of loads that can lead to reduce the power factor of a system.

1. Reactive loads - reactive loads will increase the phase angle of the fundamental voltage and current waveforms which will reduce the displacement power factor. This can be improved by adding capacitor banks to locally provide the reactive power needed by the load.
2. Nonlinear loads - According to the equation 20) current harmonics in the system contributes to the power factor reduction. This can't be improved by adding capacitor banks. Only solution to improve the distortion power factor is by eliminating current harmonics by adding active or passive filters.

## Explanation for Kekirawa pump station Issue

1. Why the power factor regulator showed the power factor as 0.99 ?

Actually the Power factor regulator is reading the displacement power factor disregarding the contribution from the distortion power factor. This is because they are designed to measure and regulate the displacement power factor only instead of true power factor, as capacitor banks are only able to improve the displacement power factor. In our case Altivar61 VSD drive reduces the phase angle difference of the fundamental waveforms of voltage and current down to around zero improving the displacement power factor up to 0.99 which can also be verified by the Graph 2. Therefore Lovato - DCRK 12 regulator reads cosø as 0.99.


## Graph 2. - Phase angle between voltage and current waveforms is nearly zero

2. Why the Power analyzer reads true power factor as 0.87 ?

Power analyzer reads the true power factor considering the contribution by both displacement and distortion power factors
As per our readings THDi is $53.21 \%$.

$$
\begin{aligned}
& p f(\text { distortion })=\frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}=\frac{1}{\sqrt{1+\left(\frac{53.21}{100}\right)^{2}}}=0.88 \\
& p f(\text { displacement })=\frac{P_{1 \text { avg }}}{V_{1 r m s} I_{1 r m s}}=\operatorname{Cos} \emptyset=0.99 \\
& p f(\text { true })=p f(\text { displacement }) \cdot p f(\text { distortion })=0.87
\end{aligned}
$$

3. Why the power factor correction bank is not effective?

As shown in the above calculations the main component responsible for the reduction of power factor is the distortion power factor. This means the reason for the power factor reduction is the existence of harmonic currents in the system. This cannot be corrected by adding capacitors.

## Proposed solution for Kekirawa pump station Issue

Proposed solution for the problem is to add 132 kW passive harmonic filters at Motor Control Panel to reduce the harmonic level down to around $5 \%$ at full load. This will improve the true power factor up to 0.98

$$
\begin{aligned}
& p f(\text { distortion })=\frac{1}{\sqrt{1+\left(\frac{T H D i}{100}\right)^{2}}}=\frac{1}{\sqrt{1+\left(\frac{0.05}{100}\right)^{2}}}=0.99 \\
& p f(\text { displacement })=\frac{P_{1 \text { avg }}}{V_{1 r m s} I_{1 r m s}}=\operatorname{Cos} \varnothing=0.99 \\
& p f(\text { true })=p f(\text { displacement }) . p f(\text { distortion })=0.98
\end{aligned}
$$

